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COSTS OF ADJUSTMENT AND THE SPATIAL PATTERN OF A GROWING OPEN CITY

BY ODED HOCHMAN AND DAVID PINES¹

This paper investigates the effect of costs of adjustment on the dynamic characteristics of intra-urban resource allocation. The analysis is concentrated on the development pattern of an open city within a system of many cities both in steady-state and in variable-state economies. Competitive equilibrium of the urban system as a whole is also discussed and compared to Pareto optimum allocation. Some of the results that characterize the resource allocation under static analysis are reestablished for the dynamic case as well. But others that follow from comparative statics are shown to be incorrect under the dynamic analysis.

1. INTRODUCTION

THIS PAPER INVESTIGATES the effect of costs of adjustment on the dynamic characteristics of intra-urban resource allocation. The analysis is mainly concentrated on the development pattern of a single city within a system of many cities, both in steady-state and in variable-state economies. Competitive equilibrium of the urban system as a whole is also discussed and compared to Pareto optimum allocation. In elaborating on the competitive allocation, a distinction is made between perfect foresight and naive expectations of the urban developers.

The dynamic characteristics of urban development were almost entirely ignored in the early urban models, which were mainly concerned with long-run analysis. By assuming that housing is produced by land only (Alonso [1], Mirrlees [15]), or that the stock of housing is perfectly malleable (Muth [17]), the difficulty associated with the durability of the housing stock was bypassed. More recently, the implications of this durability of the housing stock has become a focal point of interest (Muth [18], von Rabenau [21], Cummings and Schultze [8], and Robson [20]). However, many of these papers have ignored the spatial aspect. Those few studies which have tried to incorporate both the dynamic and spatial aspects, have adopted a putty-clay approach (see Bliss [5]), allowing the stock to change, if at all, only after it is fully depreciated; the resulting pattern of change is waves of developments, as suggested by Muth [18] and Brueckner [6].

In a recent paper (Hochman and Pines [13]) we allowed for continuous increase of stock, with adjustment costs. We used a very simple normative model, in which the cost of accommodating a population of predetermined size that increases through time, was minimized. To simplify the analysis, we also assumed constant per capita consumption of housing services through time and through space. Furthermore, we did not allow the stock of housing to decrease in any area of the city. These constraints place our earlier work in the category of irreversible-investment models (see Arrow and Kurz [3]).

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The purpose of this paper is to enrich the economic content of that model by emphasizing its positive aspect, and to remove the above-mentioned restrictions by allowing substitution between housing and other goods,² and allowing decline of the stock. The analysis deals with the development of an open city³ within a system of many cities. Workers are attracted to the city when they can obtain there at least as high a level of utility as they can obtain elsewhere. Conceptually, then, our model is very similar to that of von Rabenau [21], and Cummings and Schultze [8]. Like von Rabenau's and unlike Fujita's [9] model, it allows both increase and decrease of the housing stock. But, like Fujita we differ from von Rabenau and Cummings and Schultze in our focus on the spatial aspect of the development process. Von Rabenau ignores this aspect and aggregates housing across locations; Cummings and Schultze ignore the housing sector entirely.

Our city has a dimensionless central business district (CBD), where city residents are employed by an industry whose only input is labor and whose output is a composite good which can be consumed, exported, and used as an input in the housing industry. Each laborer belongs to a single household. Households are situated in residential zones which are characterized by the time it takes to travel from the zone to the CBD. Households are utility maximizers, consuming, at each point in time, housing services and the composite good. All households are identical in tastes, skills, and welfare levels.

Non-resident landlords, maximizers of the present value of net gains, construct housing stock in the residential zones and rent it to laborers' households. The housing stock is produced at a rate dependent on the quantity of composite good invested and the level of existing stock of housing.

We relate the development pattern of the typical city to the state of the economy. We say that *the economy* is in a steady state if the rate of return, travel time, labor productivity, and the current utility level of residents do not change with time. We say that *the city* is in a stationary state if in each zone the stock of housing and the price of housing do not change in time. (It is possible that the economy as a whole is in a steady state and yet the city is not in a stationary state.)

Our analysis shows that the optimal development pattern of a typical city which is developed from scratch may be characterized as follows:

1. At any given stage of the development, density will decline with distance from the center as will housing rents and housing values.
2. If the economy as a whole is in a steady state, the housing stock increases continuously from the very beginning of its development. The rate of increase declines through time and converges to zero in the stationary state. The quantity of resources devoted to construction declines continuously. In the stationary state this quantity becomes just sufficient to prevent the decline of the housing stock.

²This extension is a dynamic analogue of Alonso's [1] and Muth's [17] extensions of Mohring and Harwitz' [16] fixed-proportion model in the static case.

³A city is said to be "open" if people are able to migrate freely and costlessly to and from it. In this case the utility level of the population is fixed at its economy-wide level, while the city population size adjusts. In a closed city, population size is fixed and the utility level adjusts.

In the process of development each zone is gradually transformed from a net importer of resources to a net exporter.

3. If the economy as a whole is in a variable state, then the development pattern depends on the expectation formation regime. If the economy is characterized by a decline of either the utility level or of the interest rate, the housing stock will still grow continuously, but the other characteristics of development specified in 2 will not necessarily be valid. If the economy is characterized by an increase either in the utility level or in the interest rate and if the developers are myopic, the housing stock may begin to decline (unlike the pattern described in 1 above) after the early stages of growth, and may eventually even be abandoned altogether. The effect of an increase in the accessibility cost of the zone or a decrease in the productivity of producing the non-housing output is similar to that of an increase in the interest rate.

4. An increase in utility coupled with an improvement in the transportation system, especially in the outer rings, is likely to result in the eventual deterioration of the city center and in the development of the suburbs.

In addition, it is shown that:

5. In the case of perfect foresight, the present value of the net export of the city (surplus) is fully capitalized in the land values plus the value of the existing stock of housing. (The benefit of a project can therefore be measured by the increase in the value of the housing stock and land values in the area.) In the case of perfect foresight, the competitive allocation is consistent with the Pareto optimum.

The plan of the paper is as follows: Section 2 presents the model, its underlying assumptions, and the equilibrium conditions, as well as the relations between the surplus of the city and the real estate values, and between the competitive allocation and the Pareto optimum. Section 3 discusses the development pattern of a given zone when all exogenous variables are constant. Section 4 is devoted to an analysis of the spatial growth pattern. The effect of changes in the exogenous variable on the development pattern is the subject of Section 5. Reservations and possible extensions of the model are discussed in Section 6.

2. THE MODEL

The model is designed to characterize long-run spatial development in a small city within a system of open cities under a competitive regime. The cities are open in the sense that households move costlessly between any pair of cities and have perfect knowledge of the wages and prices prevailing in each city of the system.

The urban population is composed of homogeneous households (in terms of both preferences and initial holdings). Each household supplies one unit of labor services per period. Land and housing capital stock are owned by absentee landlords.⁴

⁴The assumption of absentee landlords is convenient but not consequential to the results (see Section 6).

The city is monocentric. In the center there is an export-import terminal, where the production of a composite good is concentrated. It is surrounded by residential zones. Each residential zone is denoted by i ; $i = 1, 2, \dots$, such that zone $i = n$ is closer to the center than $i = n + 1$. This spatial layout can be explained by the extremely high cost of shipping the composite good within the city, or by the nuisance created by the production process and the existence of minimum environmental standards.⁵

The spatial pattern of the city generates commuting costs. Let $T_i(t)$ units of a composite good be required to transport a worker from residential zone i to the center, where he works, in period t . Commuting costs increase with distance. Thus, $T_n(t) < T_{n+1}(t)$. It is assumed that the cost of transportation is independent of the volume of traffic (no congestion effect).⁶

A unit of labor in period t produces $w(t)$ units of the composite good. Thus, total production of the composite good in the city equals $w(t)N(t)$, where $N(t)$ is the size of the city population in period t .⁷

Since the city is small, the welfare level, $u(t)$, attainable elsewhere in the country, is exogeneously determined. Costless migration implies that households are attracted to the city only if they can achieve there the utility level $u(t)$. The utility level of a household depends on its consumption of housing services $B_i(t)$, and of a composite good $Z_i(t)$.⁸ The consumption of housing services is measured by the quantity of housing stock (e.g. square feet of net floor space) occupied per period. The relative price of housing services,⁹ in terms of the composite good, $P_i(t)$, depends on the location of the house. Hence, given the utility level, $u(t)$, the compensated demands are

$$Z_i(t) = z(P_i(t), u(t)) \quad \text{and}$$

$$B_i(t) = b(P_i(t), u(t)).$$

A household earns its marginal productivity, $w(t)$, and spends its income on the composite good $Z_i(t)$, housing services $P_i(t)B_i(t)$, and transportation $T_i(t)$. Thus, its budget constraint is¹⁰

$$(1) \quad z(P_i(t), u(t)) + P_i(t)b(P_i(t), u(t)) + T_i(t) = w(t).$$

⁵We avoid the more realistic explanation—economies of scale and agglomeration—as these imply that the optimum city cannot be sustained competitively. The concluding section discusses the extension of the model to incorporate these aspects.

⁶As this assumption is unrealistic, further extensions of the model should consider this point, as well as the investment in improving transportation (see Section 6).

⁷The implications of the model are unchanged if we assume that the composite good is produced by a linear homogeneous function of labor and capital and that the supply of capital is infinitely elastic.

⁸The composite good consumed may be different from that produced locally. For example, the city may specialize in an export composite good, the demand for which is infinitely elastic. The prices of both are assumed to be unitary. This can explain the assumption that the shipment of the composite good produced is extremely expensive while the shipment of the composite good consumed is not, and thus justify the concentration of production.

⁹Note that rent and price of housing services are considered synonymous.

¹⁰Note that the budget constraint implies that the city residents live on their current income and all landlords are absentees. Relaxation of this assumption is discussed in Section 6.

The total supply of housing services at any given period is equal to the housing stock $H_i(t)$. The change in the latter is the net construction. The production function of net construction (net addition to the stock) is assumed to be homogeneous of first degree in the stock $H_i(t)$, the composite good input $C_i(t)$, and a predetermined supply of land $L_i(t)$.¹¹ Accordingly,

$$(2) \quad \dot{H}_i(t) = h(H_i(t), C_i(t), L_i(t)),$$

$$(3) \quad L_i(t) = L_i,$$

where a dot denotes differentiation with respect to time and the parameter L_i is the fixed quantity of land in zone i .

We assume:

$$(4a) \quad h_1 < 0, \quad h_{11} \leq 0;$$

$$(4b) \quad h_2 > 0, \quad h_{22} < 0, \quad h(0, C, L) \rightarrow \bar{h} \quad \text{as} \quad C \rightarrow \infty;$$

$$(4c) \quad h_3 > 0, \quad h_{33} < 0 \quad \text{for} \quad C > 0;$$

$$(4d) \quad h_{12} < 0, \quad h_{13} < 0, \quad h_{23} > 0;$$

$$(4e) \quad h(\cdot) > 0 \quad \text{for} \quad H_i(t) = 0, \quad C_i(t) > 0;$$

$$(4f) \quad h(\cdot) < 0 \quad \text{for} \quad H_i(t) > 0, \quad C_i(t) = 0; \quad \text{and}$$

$$(4g) \quad h(\cdot) = 0 \quad \text{for} \quad H_i(t) = C_i(t) = 0,$$

where h_j denotes differentiation of $h(\cdot)$ with respect to the j variable.

Thus, our model allows for decline, expansion, and maintenance of the housing stock at a constant level. The marginal rate of depreciation, $(-\partial \dot{H} / \partial H)$, is $(-h_1)$. We assume that this rate is a nondecreasing function of the stock. The marginal productivities of land and the composite good are assumed to be decreasing. The marginal productivity of the composite good is assumed to be decreasing with the housing stock, reflecting the increasing difficulty of adding new units of housing as density increases.¹²

Now, consider a landlord who owns all land and buildings in zone i and is both a developer and a supplier of housing services there. Given the initial stock of housing, $H_i(\tau)$, the expected rates of discount, $r(t)$, and prices, $P_i(t)$ for $t \geq \tau$,

¹¹ According to this specification the rate of change of the stock is independent of age. This may perhaps be realistic with respect to brick and concrete houses but not with respect to frame houses.

¹² The function $h(\cdot)$ can be considered a transformation function. Thus, $h_1 < 0$ and $h_{11} \leq 0$ reflect the weak concavity of the function. In a conventional growth model, the relation between \dot{H} and h is linear. In our model, the relation is generalized to take into account the adverse effect of density on maintenance, congestion, and pollution costs (for a more detailed discussion see Hochman and Pines [13]). These effects are also reflected in $h_{21}, h_{31} < 0$. The presence of the costs of adjustment is reflected in $h_{22} < 0$. The conventional rationale for the costs of adjustment (see Lucas [14]) is relevant to our case as well. Additional arguments for including adjustment costs are elaborated in our earlier paper.

the landlord maximizes over $H_i(t)$ and $C_i(t)$:

$$(5) \quad V(H_i(\tau)) = \int_{\tau}^{\infty} [H_i(t)P_i(t) - C_i(t)] e^{-r(t)(t-\tau)} dt$$

subject to (2), (3),

$$(6) \quad H_i(\tau) = H_i$$

and

$$(7) \quad C_i(t) \geq 0.$$

The necessary conditions for the maximization problem are (in abbreviated notation):

$$(8a) \quad \beta_i h_2 \leq 1,$$

$$(8b) \quad (\beta_i h_2 - 1)C_i = 0, \quad \text{and}$$

$$(9) \quad \dot{\beta}_i = r\beta_i - P_i - \beta_i h_1,$$

where β_i is the costate variable of H_i , and the transversality conditions:

$$(10a) \quad \lim_{t \rightarrow \infty} e^{-rt}\beta_i(t) = 0,$$

$$(10b) \quad \lim_{t \rightarrow \infty} e^{-rt}\beta_i(t)H_i(t) = 0.$$

Obviously, $V(H_i(\tau))$ is the value of the property, including the land and the structure, to its owner. The costate of H_i , β_i , measures the marginal contribution of H_i to the value of the property, i.e., $dV_i(H_i(t))/dH_i(t)$ where H_i is evaluated at the optimum. It is therefore the shadow price of the housing stock. Accordingly, equations (8a) and (8b) state that the marginal revenue product of the composite good input, C_i (in terms of the value of the property per unit of input) does not exceed its price (= 1), and is equal to the price of the composite good if the composite good input is positive.

Rearranging, (9) may be reduced to

$$(9') \quad P_i + \dot{\beta}_i = r\beta_i - \beta_i h_1.$$

The left-hand side of (9') is the current income of a unit of housing in zone i . This includes the housing rent, P_i , and capital gains, $\dot{\beta}_i$. The right-hand side is the cost of capital per unit of housing, $r\beta_i$, plus the depreciation, $-\beta_i h_1$. Thus, equation (9') says that at the optimum the marginal cost equals the marginal revenue.

The dwelling unit size is assumed to be flexible, i.e., change in the size of the dwelling units within the building is costless. Accordingly, equality between the

supply and demand for housing implies

$$(11) \quad b(P_i(t), u(t))N_i(t) = H_i(t),$$

where $N_i(t)$ is the number of households (workers) living in zone i at period t . Let $N(t)$ be the total labor force and the total number of households in the city at time t ,

$$(12) \quad \sum_i N_i(t) = N(t).$$

Given the (exogenous) utility level, $u(t)$, and productivity, $w(t)$, the price of housing, $P_i(t)$, is determined for each zone by (1). Given $P_i(t)$ and the stock of housing, $H_i(t)$, the number of households accommodated in each zone and in the city as a whole is determined by (11) and (12).

The change in the housing stock depends on the expectation formation regime. Two extreme regimes are considered in the following discussion: myopic or naive expectations, and perfect foresight. According to the first, the developer considers the current price and discount rate to prevail forever. He continuously modifies his expectations and therefore his plan, in accordance with the changes in the current price and interest rate. In the second case, Bellman's optimization principle implies that the plan does not change with time.

In the case of perfect foresight, some basic relations can be established between the surplus of the city (to be defined hereafter) and city real-estate values, and between the competitive allocation and Pareto optimum allocation.

The total production of the composite good by the city work force $X(t)$, is:

$$(13) \quad X(t) = N(t)w(t).$$

The city's current surplus or exports, $S(t)$, is

$$(14) \quad S(t) = X(t) - \sum_i \{N_i(t)[z(P_i(t), u(t)) + T_i(t)] + C_i(t)\}.$$

It follows from (1), (11), (12), (13), and (14) that

$$(15) \quad S(t) = \sum_i (H_i P_i - C_i).$$

Hence, multiplying both sides by e^{-rt} , integrating between τ and ∞ , and summing over i , we have:

$$(16) \quad \sum_i V(H_i(\tau)) = \int_{\tau}^{\infty} e^{-r(t-\tau)} S(t) dt,$$

which may be formulated as follows:

PROPOSITION 1: *If landlords have perfect foresight, the market value of the land and the structures of the city in any period τ is equal to the present value of the surplus that would be realized between τ and ∞ .*

Furthermore, we have the following:

PROPOSITION 2: *If landlords have perfect foresight, the market equilibrium allocation maximizes the present value of the city surplus $S(t)$ over $P_i(t), C_i(t), H_i(t), N_i(t), N(t), X(t)$, for all $i = 1, 2, \dots$, subject to (2), (3), (6), (7), (11), (12), and (13).*

PROOF: The necessary conditions of the optimization problem specified above are identical to the equilibrium conditions (1), (2), (3), (6), (7), (8a), (8b), (9), (10a), (10b), (11), (12). *Q.E.D.*

Now define a competitive allocation in the economy as a whole as one where every worker is accommodated in one of the predetermined potential city sites. Formally, the above competitive allocation is defined by (1), (2), (3), (6), (7), (8), (9), (10), (11), and

$$(17) \quad \sum_{ij} N_i^j(t) = \bar{N}(t), \quad 0 \leq t < \infty,$$

where N_i^j is the population of zone i in city j ($j = 1, \dots, J$). Accordingly, $u(t)$ is endogenously determined. Using this definition we have the following:

PROPOSITION 3: *Suppose that landlords have perfect foresight. Then:*

(a) *Given the equilibrium $u(t)$, the equilibrium allocation maximizes the present value of the surpluses of all the cities.*

(b) *If the pure rate of time preference of a resident of the city equals the interest rate (i.e., the intertemporal utility level of a resident at time τ is given by $\int_{\tau}^{\infty} e^{-rt} u(t) dt$) and if the welfare level of landlords is an increasing function of the present value of the total surpluses of the cities (i.e., the value of their properties), then the competitive equilibrium is Pareto optimal.*

PROOF: (a) Add a superscript j to denote a potential city. Then, consider the maximization of $V^* = \sum_j \int_0^{\infty} e^{-rt} S^j(t) dt$ over $p_i^j(t), C_i^j(t), H_i^j(t), L_i^j(t), N_i^j(t), N^j(t), X^j(t)$ subject to (2), (3), (6), (7), (11), (12), (13), and (14) where $u(t)$ is evaluated at its equilibrium value.

Relations (1)–(3), (6), (8)–(13), which describe the competitive equilibrium, can be shown to be consistent with the necessary conditions of the above maximization problem. It follows that the solution to the above maximization problem is identical to the competitive allocation. In particular, the solution for $N_i^j(t)$ is identical to the corresponding equilibrium function of time. Thus, we can use (17) as an additional constraint without changing the solution to the maximization problem. Consequently, given the equilibrium utility $u(t)$ and the predetermined population size $\bar{N}(t)$, the present value of the total urban surplus, V^* , is maximized as argued in the proposition.

Part (b) follows from part (a) (see Hochman [11]).

3. THE GROWTH PATTERN OF A GIVEN ZONE
WHEN THE ECONOMY IS IN STEADY STATE

In this section we assume that the economy as a whole is in a steady state, such that the utility $u(t)$ and the discount rate $r(t)$ are constant. We also assume that the transportation costs, $T(t)$, and productivity $w(t)$ do not vary with time. Thus, $\dot{u}(t) = \dot{w}(t) = \dot{r}(t) = \dot{T}(t) = 0$. In this case the plans of a landlord with naive expectations and of one with perfect foresight coincide.

We shall derive the growth pattern in a given zone by examining the phase diagram corresponding to the necessary conditions considered in the previous section. As we shall show, the properties of the qualitative solution can be derived once we ascertain the position of $C = \text{constant}$, $\dot{H} = 0$, and $\dot{\beta} = 0$ in the (H, β) plane and draw the phase diagram. We start by determining the positions of $\dot{H} = 0$ and $\dot{\beta} = 0$ and possible trajectories, to satisfy the conditions (2), (3), (4), (8), and (9).

Differentiating (2) with $\dot{H} = 0$, using (8a) as an equality, and using (4a), (4b), (4d) yields

$$(18) \quad \left. \frac{d\beta}{dH} \right|_{\dot{H}=0, C=0} = \frac{\beta(h_1 h_{22} - h_2 h_{21})}{h_2^2} > 0.$$

At the intersection of $\dot{H} = 0$ and the β axes, $C = 0$. This follows from (2) with $\dot{H} = 0$, (4e), and (4g). Thus, from (8a),¹³

$$(19) \quad \beta|_{\dot{H}=H=0} = \frac{1}{h_2(0, 0, L)}.$$

Differentiating (8a) as an equality, assuming that the second order (sufficient) condition $h_{11} - h_{12}^2/h_{22} < 0$ is satisfied,¹⁴ and using (4a), (4b), and (4d) yields:

$$(20) \quad \left. \frac{d\beta}{dH} \right|_{\dot{\beta}=0, C>0} = \frac{\beta(h_1 h_{22} - h_{12}^2)}{(r - h_1)h_{22} + h_{12}h_2} < 0.$$

Also, for $C = 0$, we have by (9) and (4a):

$$(21) \quad \left. \frac{d\beta}{dH} \right|_{\dot{\beta}=0, C=0} = \frac{\beta h_{11}}{r - h_1} \leq 0.$$

The above results allow us to prove:

¹³It should be noted that the value of $\beta|_{\dot{H}=H=0}$ is not uniquely defined by (2) and (8a). However, our interest is in the upper limit of all the values of β belonging to this set, which is also the limit of $\beta(H)|_{\dot{H}=0}$ when H approaches zero. Only this limit fulfills (19) and henceforth only this value is meant whenever $\beta|_{\dot{H}=H=0}$ is discussed. From (2) and (8a) it follows that strict equality in (8a) holds in this case, even though C is zero.

¹⁴The sufficient condition $h_{11} - h_{12}^2/h_{22} < 0$ states that the Hamiltonian of the optimization problem, evaluated at the optimal value of C , is a concave function of H ; for given values of β . For a proof of the sufficiency of this condition see Arrow and Kurz [3, Ch. II].

LEMMA 1: (a) If $p > [r - h_1(0, 0, L)]/h_2(0, 0, L)$, the loci $\dot{H} = 0$, $\dot{\beta} = 0$ intersect within the positive quadrant, at a finite value of H and β , and (b) if $p < [r - h_1(0, 0, L)]/h_2(0, 0, L)$, the two loci do not intersect in the positive quadrant.

PROOF: Define:

$$\mu(C) = [r - h_1(0, C, L)]/h_2(0, C, L).$$

Then, from (4b) we have:

I. $\lim_{C \rightarrow \infty} \mu(C) = \infty,$

and by (4):

II. $\frac{\partial \mu(C)}{\partial C} = -\frac{h_{12}}{h_2} - \frac{rh_{22}}{h_2^2} + \frac{h_1 h_{22}}{h_2^2} > 0.$

Let \tilde{C} be the value of C at the intersection of the loci $\dot{\beta} = 0$ and $H = 0$. Then, it follows from (8a) and (9) that:

III. $P \leq [r - h_1(0, \tilde{C}, L)]/h_2(0, \tilde{C}, L).$

In part (a) we have:

IV. $P > [r - h_1(0, 0, L)]/h_2(0, 0, L).$

I, II, and IV imply that III can be satisfied only if $\tilde{C} > 0$. Thus, (4b) and (8b) yield:

$$\beta|_{\dot{\beta}=H=0} = \frac{1}{h_2(0, \tilde{C}, L)} > \frac{1}{h_2(0, 0, L)} = \beta|_{\dot{H}=H=0}.$$

The above chain, together with (18) and (4a), prove part (a) of Lemma 1.

In part (b) we have:

V. $P < [r - h_1(0, 0, L)]/h_2(0, 0, L).$

If $\tilde{C} > 0$, then, by (8b):

VI. $P = [r - h_1(0, \tilde{C}, L)]/h_2(0, \tilde{C}, L),$

but then II, V, and VI imply a contradiction. Thus, $\tilde{C} = 0$, and, therefore:

$$\beta|_{\dot{\beta}=H=0} \leq \frac{1}{h_2(0, 0, L)} = \beta|_{\dot{H}=H=0}.$$

This chain, (18), and (21) yield part (b) of Lemma 1.

Q.E.D.

Differentiating (2), (9), and (8a) as an equality (for $C > 0$), and using (4), we obtain:

$$(23a) \quad \left. \frac{d\beta}{d\beta} \right|_{dH=0, C>0} = r - h_1 + \frac{h_{12}h_2}{h_{22}} > 0,$$

$$(23b) \quad \left. \frac{d\beta}{d\beta} \right|_{dH=C=0} = r - h_1 > 0,$$

$$(23c) \quad \left. \frac{d\dot{H}}{d\beta} \right|_{dH=0, C>0} = -\frac{h_2^2}{h_{22}} > 0, \quad \text{and}$$

$$(23d) \quad \left. \frac{d\dot{H}}{d\beta} \right|_{dH=C=0} = 0.$$

The above analysis implies that for a zone i , where $P_i > [r - h_1(0, 0, L_i)]/h_2(0, 0, L_i)$, the possible trajectories consistent with (2), (3), (4), (8), and (9) are I–VI in Figure 1. However, relations (4a), (4b), and (4d) imply that H is bounded from above and $h(\)$ is concave in H and C . Therefore, extending Cass' Theorem [7] to our problem, it follows that the optimal paths are along the stable arms I and II. Thus, Lemma 1 and the above results imply:

LEMMA 2: *Let there be an initial condition, $H_i(\tau) = H_{i\tau} \geq 0$. Then the optimal path starts at $H_{i\tau}$ and follows the stable arm (I or II) towards the stationary state, β^∞, H^∞ .*

If

$$P_i > \frac{r - h_1(0, 0, L_i)}{h_2(0, 0, L_i)},$$

then, according to Lemma 1, $\beta^\infty, H^\infty > 0$. If

$$P_i \leq \frac{r - h_1(0, 0, L_i)}{h_2(0, 0, L_i)},$$

$H(\infty) = 0$, i.e., if $H(\tau) = 0$ the optimal strategy is to do nothing; if $H(\tau) > 0$, the optimal strategy is to let the stock decline to 0.

It follows from (1) that P_i declines with T_i . Thus, the boundary of the city is in the first zone where T_i is so large that

$$P_i \leq \frac{r - h_1(0, 0, L_i)}{h_2(0, 0, L_i)} \quad \text{and therefore} \quad H^\infty = 0.$$

The boundary can be at the smallest T_i . In this case either from the start or eventually there will be no city.

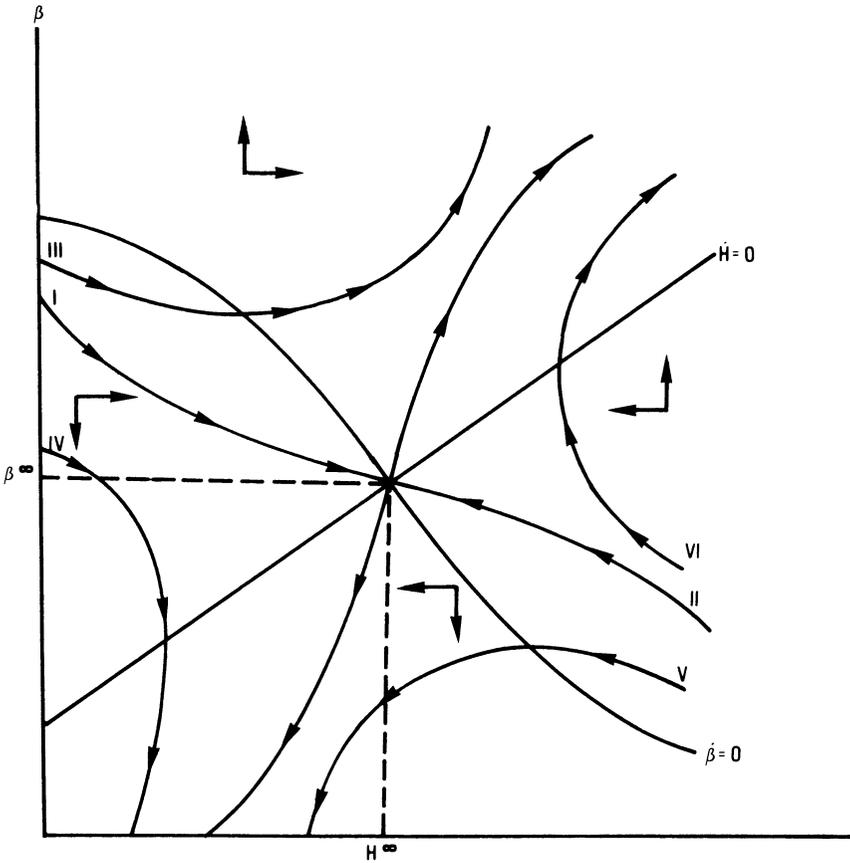


FIGURE 1.

Now consider the loci of all β and H which satisfy $\beta h_2 = 1$ for a given $C = \bar{C} = \text{constant} \geq 0$. Differentiating (8a) as an equality yields:

$$(24) \quad \left. \frac{d\beta}{dH} \right|_{C=\bar{C}} = - \frac{\beta h_{21}}{h_2} > 0.$$

It intersects the β axis at $1/h_2(0, \bar{C}, L)$. In particular, if $\bar{C} = 0$, it intersects the β axis where the locus $\dot{H} = 0$ does. A comparison of (24) with (18) implies that, for a given H , the slope of the locus $C = 0$ is smaller than that of $\dot{H} = 0$.

Also, by (8a) as an equality:

$$(25) \quad \left. \frac{d\beta}{d\bar{C}} \right|_{H=\text{constant}} = - \frac{\beta h_{22}}{h_2} > 0.$$

Equations (24) and (25) are used in Figure 2.

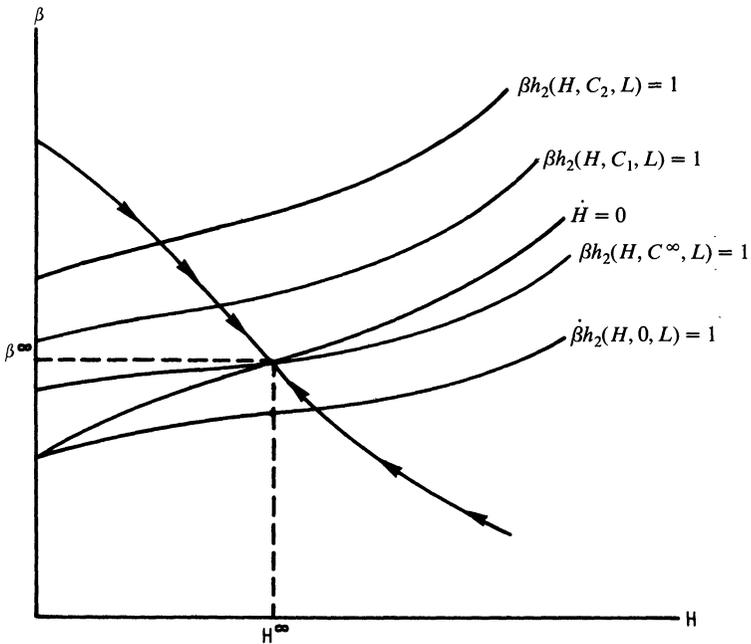


FIGURE 2.

The implications of the analysis of this section are summarized in the following proposition:

PROPOSITION 4: For a zone *i* where

$$P_i > \frac{r - h_1(0, 0, L_i)}{h_2(0, 0, L_i)},$$

- (a) $H_i(\tau) \leq H_i^\infty \Leftrightarrow \dot{C}_i(t) \leq 0$ for $t \geq \tau$,
- (b) $H_i(\tau) \leq H_i^\infty \Leftrightarrow d[H_i(t)P_i - C_i(t)]/dt = \dot{S}_i(t) \geq 0$, for $t > \tau$.
- (c) If $H_i(\tau) = 0$, then $S_i(\tau) < 0$,
- (d) $S_i^\infty \geq 0$,
- (e) $\dot{V}_i(H_i(t)) \geq 0 \Leftrightarrow H_i(t) \leq H_i^\infty$, and $V_i(H_i(t)) > 0$ for all $t \geq 0$,

where $S_i = N_i(w_i - z(P_i, u(t)) - T_i) - C_i$.

PROOF: Implication (a) follows from Figure 2. Implications (b) and (c) follow from Figure 2 and the analysis which implies equation (15).

Applying Euler's Theorem to (2) for $\dot{H} = 0$, using (9) for $\dot{\beta} = 0$, (8b) and the analysis which yields (15), it follows that:

$$S_i^\infty = \beta^\infty (H_i^\infty r + h_3 L_i) \geq 0,$$

which proves (d).

Let $\mathcal{H}(t)$ be the current value Hamiltonian of our maximization problem, i.e. $\mathcal{H}_i(t) = [H_i(t)P_i(t) - C_i(t)] + \beta_i(t)\dot{H}_i(t)$; then, following Arnott, Davidson and Pines [2]:

$$V_i(t) = (\mathcal{H}_i(t)/r) = \frac{1}{r} [H_i P_i - C_i + \beta_i \dot{H}_i],$$

which upon differentiation (using (9)) yields:

$$\dot{V}_i = \dot{H}_i \beta_i + (\dot{C}_i (\beta_i h_2 - 1)/r).$$

(8a) and (8b) imply that for $\dot{C} = C = 0$ and $C > 0$ the second term must vanish. Thus, since $\beta_i > 0$, and $\dot{H}_i \geq 0 \Leftrightarrow H_i \leq H_i^\infty$, the first part of (e) is proved.

As to the second part, it is clear that if $H_i(t) > H_i^\infty$, even a "do nothing" policy implies positive profits. *A fortiori* the optimal policy must yield positive profits, which means $V_i(H_i(t)) > 0$. If $H_i(t) < H_i^\infty$, we have only to show that for $H_i = 0$, $V_i(0) > 0$ (since, as already shown, $\dot{V}_i > 0$).

Using $V(H_i(t)) = (1/r)\mathcal{H}(t)$, $H_i(t) = 0$, (8a) as an equality, and Euler's Theorem for $h(\cdot)$, we obtain:

$$V_i(0) = \beta h_3 L_i / r > 0.$$

The inequality follows from the discussion of Lemma 1 ($C(0) > 0$), and from (4c). This proves the second part of (e). Q.E.D.

It follows from implications (b), (c), and (e) that for any zone being developed from scratch there exists a housing stock level H_i^* , $0 < H_i^* < H^\infty$, where the zone ceases to be a net importer of resources and becomes a net exporter.¹⁵

4. THE SPATIAL PATTERN OF CITY GROWTH WHEN THE ECONOMY IS IN A STEADY STATE

In this section the effect of accessibility on the growth pattern is analyzed. To facilitate the analysis, we assume henceforth that the city is divided up into zones of equal area (i.e., $L_i = L$, $i = 1, 2, \dots$).

¹⁵Note that in our model borrowing and lending of the composite good by landlords for an interest rate r is possible. Thus, when a zone (or the whole city) starts to develop it is a net borrower, and in the stationary state it may be a net lender. In our type of model where intragenerational transfers are impossible, intertemporal transfers (i.e., lending and borrowing) are possible only through accumulation of property (and houses). Hence, only those who may own property can lend and borrow. In the last section of this paper we allow workers to own property (positive or negative) as well and hence benefit from intertemporal transfers. Note, however, that unlike the productive sector, decisions concerning intertemporal transfers in consumption are made at the economy-wide level and hence are exogenously given to the individual city.

LEMMA 3: *Let P^n and P^m be two levels of rent paid to landlords for housing services. If $P^n > P^m$, then the equilibrium path corresponding to P^n lies everywhere strictly above the one corresponding to P^m .*

PROOF: First, we show that $\beta^n(\infty) > \beta^m(\infty)$, and $H^n(\infty) > H^m(\infty)$, where $\beta^i(t)$ and $H^i(t)$ are the values of β and H at time t corresponding to p^i ($i = n, m$). Second, we show that the two equilibrium paths cannot intersect.

To prove the first part we differentiate (8a) and (9) as an equality, and use the results to derive

$$\frac{d\beta}{dP} \Big|_{dH = \dot{\beta} = 0} = [r - h_1 + h_2 h_{12} / h_{22}]^{-1} > 0.$$

Thus, since the locus $\dot{\beta} = 0$ shifts upward and locus $\dot{H} = 0$ does not change, the stationary state points β^∞ and H^∞ move to the right and upward on $\dot{H} = 0$.

Regarding the second part, suppose the equilibrium paths intersect in the (βH) plane. Then, it follows from (8b) that at the intersection, H and C are equal for the two paths. But then, it follows from (2) that $\dot{H}^m = \dot{H}^n$ and from (9) that $\dot{\beta}^n < \dot{\beta}^m$. This implies that the n intersects the m equilibrium trajectory from above. But in order to satisfy the first part, the n equilibrium trajectory must eventually intersect the m equilibrium trajectory from below, which is impossible.

Q.E.D.

It follows from (1) that P_i decreases with i , i.e., increases with the accessibility of the zone. Thus, Lemmas 1 and 3 imply the following proposition:

PROPOSITION 5: *If the initial stock of housing is zero, then:*

(a) *The development takes place simultaneously in all the zones that are eventually developed.*

(b) *The more accessible the zone is, the higher the level of its housing stock H at any point of time and, of course, in the stationary state.*

(c) *The more accessible the zone, the higher the value of its housing stock $V(H(t))$ (since V increases with H and P).*

5. THE CITY GROWTH PATTERN, WHEN THE ECONOMY IS NOT IN A STEADY STATE

We begin the discussion by considering the case of naive expectations. The developer is assumed to maximize:

$$V(H(\tau)) = \int_{\tau}^{\infty} [H_i(t)P_i(\tau) - C_i(t)]e^{-r(t-\tau)} dt$$

subject to (2), (3), (6), and (7).

First, consider the effect of an increase in $P_i(\tau)$ such that up to time t_0 , P_i^m prevails and thereafter P_i^n where $P_i^n > P_i^m$. The change is reflected in an upward movement of locus $\dot{\beta} = 0$ and (according to Lemma 3) an upward shift of the

equilibrium path. Thus, up to t_0 the zone is developed according to the equilibrium path corresponding to P_i^m . At t_0 , β_i jumps up to the equilibrium path corresponding to P_i^n and the development continues according to this path.

Second, suppose that P_i increases continuously, eventually converging to P^∞ , $P^\infty < \infty$. The paths I, II, III, and IV in Figure 3 are consistent with such a continuous increase in P_i .

A zone which is built from scratch is still developed continuously from the very beginning, as before, but β may now increase during some or perhaps all time intervals. Moreover, if the stock initially exceeds its stationary state level, it may first decline and then rise again towards the stationary state level (see Figure 3).

Suppose that initially the zone is not developed since P_i is too small, such that condition (a) of Lemma 1 is not satisfied. Let P_i increase so that at some point of time, t_0 , P_i reaches the level which satisfies this condition. From that time on, the zone is continuously developed.

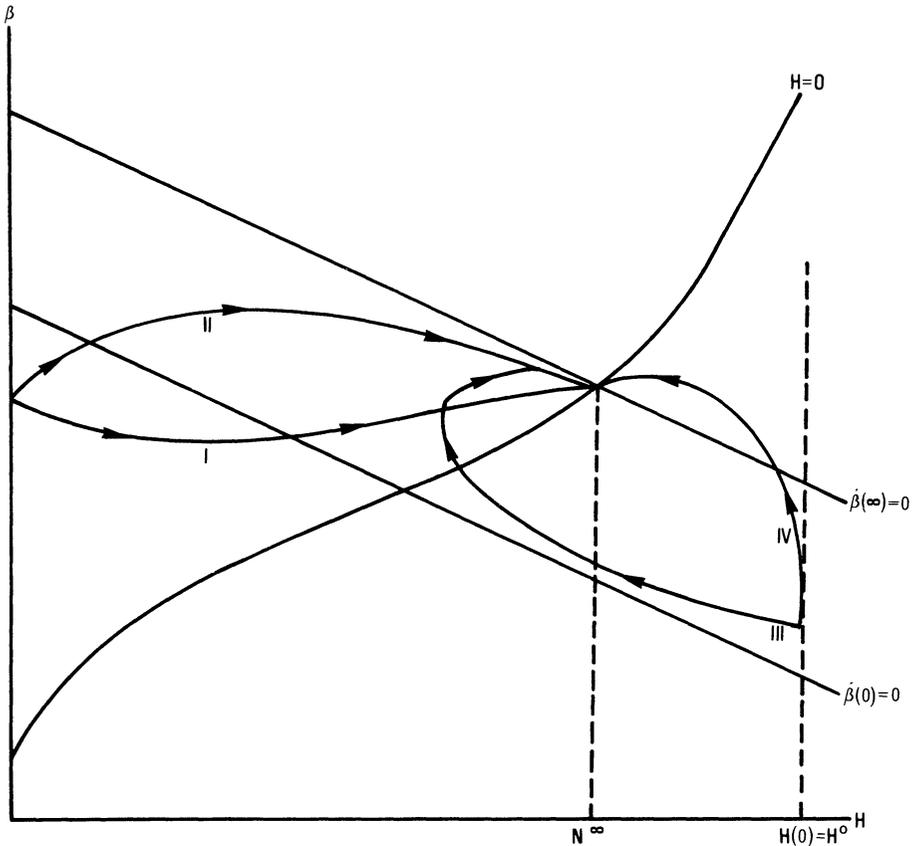


FIGURE 3.

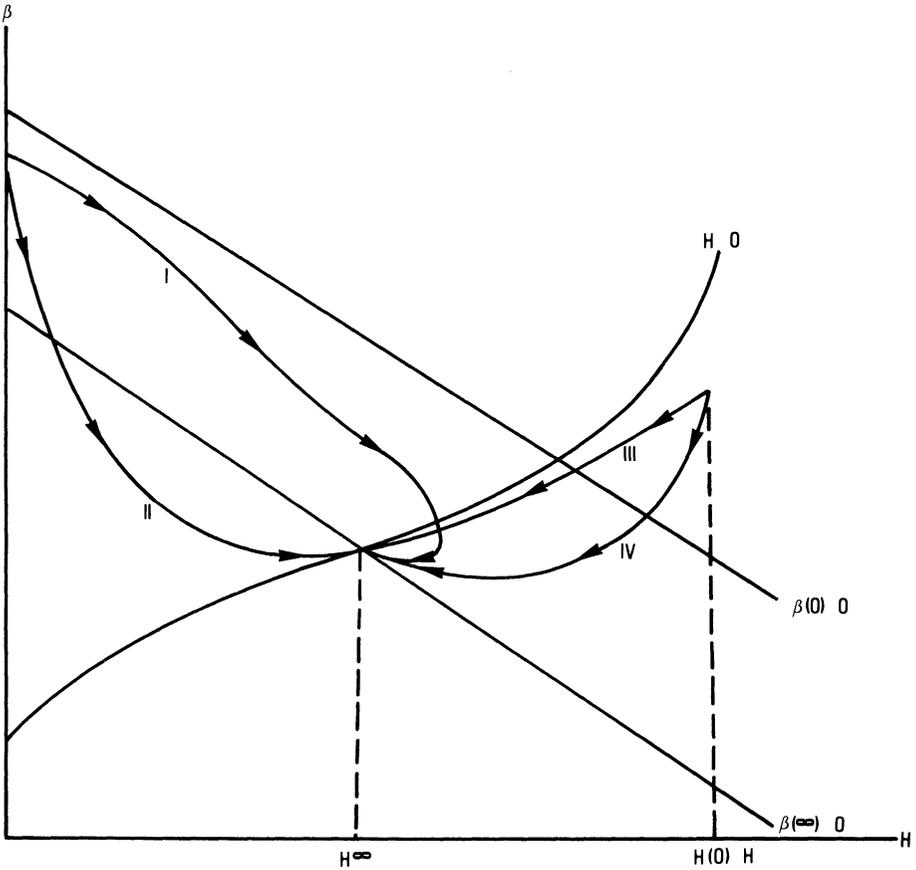


FIGURE 4.

Next, suppose that P decreases continuously and eventually converges to P^∞ . The paths I, II, III, and IV in Figure 4 are consistent with this situation. In this case it is possible that overshooting will occur, i.e., the stock, after being accumulated to some level of H higher than the stationary state level, decreases to its stationary state level. Moreover, if P_i eventually decreases below $[r - h_1(0, 0, L)]/h_2(0, 0, L)$, the stock will decline and the zone will be totally abandoned.

Now we turn to analyzing the effect of a change in r . First, consider the effect of an increase in r . Similar to Lemma 3, which refers to a change in P_i , we can show the following lemma.

LEMMA 4: *If $r^n > r^m$, then the equilibrium path corresponding to r^n lies everywhere below that corresponding to r^m .*

PROOF: For a positive H the locus $\beta(H)|_{\dot{\beta}=0}$ is determined by (8) and (9). Their differentiation yields

$$\frac{d\beta}{dr} \Big|_{dH=\dot{\beta}=0, C>0} = -\beta[(r - h_1) + h_{12}h_2/h_{22}]^{-1} < 0,$$

$$\frac{d\beta}{dr} \Big|_{dH=\dot{\beta}=0, C=0} = -\beta/(r - h_1) < 0,$$

where the inequality follows from (4a), (4b), and (4d). This implies that when r increases, the locus $\beta = 0$ in the (β, H) plane shifts downward. The locus $\dot{H} = 0$, which is independent of r , does not vary. Hence $H^n(\infty) < H^m(\infty)$ and $\beta^n(\infty) < \beta^m(\infty)$, and, as in Lemma 3, the whole path corresponding to r^n is below the one corresponding to r^m . Q.E.D.

The effect of a continuous decrease in r converging to a finite r^∞ is similar to that of a continuous increase of P_i converging to a finite P^∞ . This is reflected in Figure 3. The effect of a continuous increase in r is similar to the effect of a continuous decrease in P as reflected in Figure 4.

In comparing the optimal path (o.p.) which corresponds to perfect foresight with the one that corresponds to naive expectation, we observe that when p increases (r decreases):

(a) The stationary state (H^∞, β^∞) is the same for the two paths.

(b) For any given time period, t , $\beta(t)$ which corresponds to perfect foresight, should be above the stable arm which corresponds to naive expectation for that time period; only at infinity do they converge to the same point (H^∞, β^∞) . Otherwise, the o.p. of the perfect expectation would not be able to converge to β^∞, H^∞ as required by (a).

(c) Given H , a higher β is associated with a higher rate of growth, \dot{H} . This relation can easily be deduced from Figure 2, where, for a given H , a higher β is associated with a higher value of C , and, therefore, according to equation (2), with a higher value of \dot{H} .

Observations (a) through (c) imply that the o.p. corresponding to perfect foresight will start with a higher value β , than the one of naive expectation. Furthermore, at any given time period, the accumulated stock of housing $H(t)$, under perfect foresight regime exceeds that of naive expectation. It is possible that a developer with perfect foresight will begin developing a zone which is still left undeveloped by a myopic developer.

The opposite relations hold when $P(t)(r(t))$ is decreasing (increasing). For example, in this case a myopic developer may incur losses by developing a zone which a developer with perfect foresight would not develop at all.

So far, we have examined how changes in r and P affect the equilibrium path. But as P itself is endogenous, we must relate Figures 3 and 4 to exogenous variables such as u , w , and T . This can be done by using (1). Accordingly, P

increases when either u or T decreases, or w increases. Thus, Figure 3 corresponds to a decrease in either u or T , or an increase in w ; Figure 4 corresponds to an opposite change in these variables.

In a dynamic economy we may distinguish between two major states: (a) the steady state, and (b) a variable state approaching the steady state. The growth literature includes three basic types of dynamic models. The first is Solow's model, in which the rate of savings and regeneration are both given exogenously. In the second type, saving is endogenous but regeneration is still exogenous (Bardhan [4]). In the third type, the rate of regeneration is also determined endogeneously (Razin and Ben-Zion [20]). All these models are steady-state models with a positive rate of population growth, while the utility level and level of capital per capita (and thus the rate of return) are fixed, i.e., $\dot{u} = \dot{r} = 0$. In a single city, when $\dot{P} = 0$ the rate of population growth attains its maximum value when the city begins to develop ($\dot{N}(0)/N(0) = \infty$) and its minimum value at infinity ($\dot{N}(\infty) = 0$); in between it exhibits a monotonic decreasing function. Thus, to maintain a constant population growth throughout the economy new cities have to be developed continuously. Thus, when the economy is in a steady state, ($\dot{u} = \dot{r} = \dot{T} = 0$, which implies $\dot{P} = 0$) new cities are started continuously, and their growth pattern, characterized by $\dot{P} = 0$, is as described above.¹⁶ When the whole economy is in a variable state approaching steady state, both utility level and property value (land and housing) per capita are changing and approaching their steady state levels. In the models mentioned above, we distinguish two characteristic cases: $\dot{u} > 0, \dot{r} < 0$; and $\dot{u} < 0, \dot{r} > 0$. Thus, in an economy approaching a steady state, the net effect on P cannot be ascertained *a priori*. Therefore we cannot determine whether Figures 3 and 4, or a combination of the two, correspond to a growing economy.

We now turn to the pattern of spatial development when the economy is not in a steady state. Suppose that either the utility level or the interest rate or both decline through time and converge to a certain positive level when $t \rightarrow \infty$. Then all $P_i(t)$ are monotonously increasing functions of time. $\dot{P}_i \geq 0$, $\lim_t P_i(t) = P_i < \infty$, where P_i^∞ are the respective steady states of P_i , $i = 1, 2, \dots$. The inequality $P_i(t) > P_j(t)$ when $j > i$ holds for all t . Distant zones, which are not developed at the beginning because $P_i(0)$ is too low (see Lemma 1), are developed as time passes.

Much more intriguing is the spatial pattern of development when the utility and/or the interest rate increase through time, in which case P_i decreases through time. Hence the boundary where the development takes place moves

¹⁶In the models of a growing economy mentioned above there are no resources whose quantity is exhaustible, such as land. If there were, then a steady state with a positive growth rate would be impossible (see, for example, Hochman and Hochman [12], with the case of an undesirable local public good). Since land is a productive factor in our model, it is unlikely that such a steady state is indeed possible in an economy containing cities like ours. In this case, housing rents must vary with time; therefore our solution for $P = 0$ does not describe a real solution and is only instrumental for deriving the solutions to the other cases.

closer to the center as time passes. In the process of development, the housing stock accumulated beyond this stationary-state boundary deteriorates, decreases, and eventually vanishes. It may be abandoned even before it completely vanishes if the rent falls to zero fast enough.

This spatial pattern is not characteristic of that observed in many growing metropolitan areas. On the contrary, the center often deteriorates while the fringe is developed and the boundary moves further out. This can be explained by a simultaneous increase in u and a decrease in P_i , coupled with a decrease in T_i , especially at the outer rings. The increase in u may imply a process of growth followed by deterioration throughout the city, but the relative advantage of the outer rings in transportation improvement can counterbalance or even outweigh the negative effect of the increase in u .

Improvement of suburban roads and the increasing use of private cars has shortened commuting time from the outer zones, while commuting time from the inner zones has actually increased due to congestion caused by the increase in traffic volumes. Thus growth in suburban zones has been boosted sufficiently to offset the effects of deterioration there. No such boost has been given to the inner ring, which continues to decay and deteriorate.

6. RESERVATIONS AND EXTENSIONS OF THE MODEL

The model presented in this paper involves a number of restrictions, including the following:

1. The depreciation is age-independent.
2. The level of service of the transportation system is exogenous. Both the congestion effect and investment in improving transportation service are ignored.
3. The urban population is assumed to be homogeneous.
4. Increasing returns to scale and economies of agglomeration in the production of the composite good are ignored.
5. The city layout is unrealistic.
6. City residents are assumed to live on their current wage and all landlords are absentees.

Extending the model to remove restrictions 5 and 6 is not very difficult. We can assume, for example, that city residents earn $\Pi(t)$ nonwage income per period. This sum is exogenous to the city, since it is nationally determined (otherwise we cannot assume that the national urban population is homogeneous). The effect of exogenously determined profits is similar to that of $w(t)$; therefore its introduction does not complicate the analysis.

Extending the model to remove restriction 4 will eliminate the absolute independence of the development in a particular zone from the development of the city as a whole. Some of the conclusions regarding the relation between the competitive allocation and optimality will no longer hold. Extension of the model to include economies of scale and agglomeration can be carried out along the lines suggested for a static case by Hochman [11]. With an appropriate government subsidy, financed by a tax on land rent, the optimum allocation can be achieved. The resulting dynamic analysis will not change, except that when the

economy is in a steady state $\dot{N}(t) \geq 0 \Leftrightarrow \dot{w}(t) \geq 0$. The effect of changes in $w(t)$ was discussed in Section 5.

Much more intriguing is the extension of the model to remove restrictions 1–3. For example, with the introduction of congestion and investment in transportation, the separability of this model to independent zones is completely lost and with it much of the simplicity of the analysis. We leave this extension, among others, for separate study.

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